# Photogrammetric Rectification of Oblique Trimetrogon Imagery

# Trent Technical Note 99-1

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## INTRODUCTION

The purpose of this Technical Note is to document the procedures which were developed for quantitative analysis of an oblique air photograph showing the positions of the terminuses of White Glacier and Thompson Glacier, Axel Heiberg Island, Nunavut. This image, part of which appears as Figure 1, dates from 11 August 1948 and is probably the oldest surviving artefact containing information on the glaciers. Although its quality leaves something to be desired because of cloud cover, it was deemed worthwhile to try to extract the information in a quantitative form so as to permit objective measurement of the terminus positions. These positions have already been measured on several later images (Cogley *et al.* 1996), and the glaciological implications of the new results from Figure 1 are discussed by Cogley and Adams (1999). It is planned to take advantage of these new procedures in quantitative studies of other early oblique images of the two glaciers, and possibly of other glaciers in the High Arctic. There is a large quantity of potentially retrievable information in these early historical records of a remote part of the cryosphere.

Early air-photographic surveys of northern North America relied on trimetrogon photography to increase terrain coverage. “Trimetrogon” refers to a survey method in which three cameras, originally of the Metrogon brand, were mounted in a plane transverse to the flight path of the aircraft. One camera pointed to the nadir and one to either side of the flight path at a depression angle from the horizontal of about 30°. The method was superseded in the mid-1950s when technological advances made vertical photographs from higher altitudes a more economical source of information for cartographic purposes. Nevertheless there remains an enormous archive of trimetrogon coverage (see Dunbar and Greenaway 1956 for abundant illustrations) which may still be “mined” for objective measurements of change in the landscape. Fluctuations of glacier terminuses are but one class of such changes.

The mathematics and technology needed for photogrammetric treatment of trimetrogon imagery became well developed during the 1940s and 1950s, and many cartographic products resulted. The mathematics and technology are summarized by Imhof and Doolittle (1966). In the 1950s, of course, there was no access to computers, and mapping from trimetrogon imagery required elaborate and expensive machines to assist with the labour of transferring information from image to map by “analogue” methods. It appears that in the subsequent 40 years there has been no systematic conversion of the photogrammetric methods to digital versions which can take advantage of the power of computers. Consequently it was necessary to develop a modern algorithm for manipulation and extraction of the information in Figure 1 and similar images. This algorithm is laid out in detail below. Although it relies heavily on Imhof and Doolittle (1966), the analysis for any single oblique photograph avoids the need to work with the accompanying vertical photograph, which is used only for a first guess at the location of the nadir of the oblique photograph.

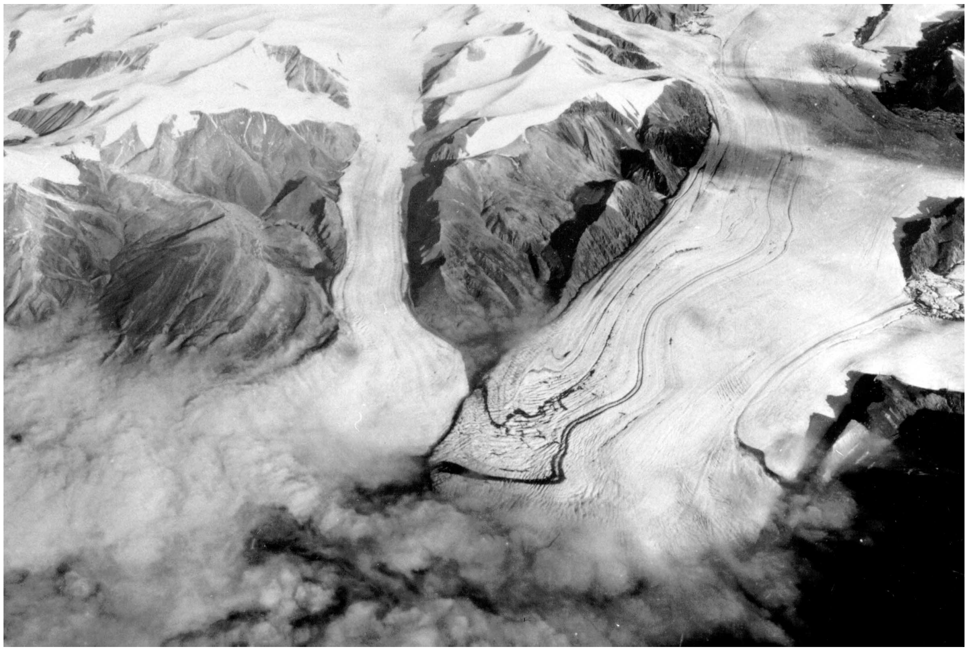

Figure 1. Part of photograph 60LT-72PL-C-8M219-72RS (11 August 1948), showing the contiguous terminuses of White Glacier (left) and Thompson Glacier (right), Axel Heiberg Island. The photograph is a left oblique taken from a flying height of approximately 6 700 m; the look azimuth is 313°, or northwest.

## PHOTOGRAMMETRIC PROCEDURES

The essence of the rectification procedure is to define the geometric relationship between image points and corresponding points on the ground by fitting a model to a set of control points. These are points whose ground coordinates are known accurately by measurement. Ideally the control points would be measured geodetically, but we obtained good results by measuring carefully chosen points on a large-scale map (National Research Council 1962a) which was itself based on reliable ground surveys (Haumann 1963). This reliance on existing ground control is a limitation of the procedure, but it is not necessarily fundamental. For example if a series of images is being studied to develop a sequence of glacier terminus fluctuations then one of the images in the sequence may be selected as a reference image and the others may be "controlled" by modelling the relationship between their image points and the corresponding points on the reference image. This is the subject of continuing algorithm development.

## Photograph Geometry

Using a transparent overlay, we first located the *principal point* (centre) $P$ of the photograph, defined the $X'$ axis running fore and aft through the principal point, defined the $Y'$ axis at right angles to the $X'$ axis, marked the trace of the horizon at several points, and located the tangent to this gently-curved trace as accurately as possible by eye (Figure 2). The tangent is called the *apparent horizon*. The positive $Y'$ axis and the perpendicular from the tangent point $A$, which passes through the principal point, determine the *swing angle* $\sigma$, which is the rotation of the photograph in its own plane about the optical axis of the camera. All subsequent work is done in $X$ and $Y$ coordinates; the $X$-$Y$ coordinate frame, in which the apparent horizon is a line on which $Y =$ constant, is derived from the $X'$-$Y'$ coordinate frame by a counterclockwise rotation through $\sigma$ degrees about P.

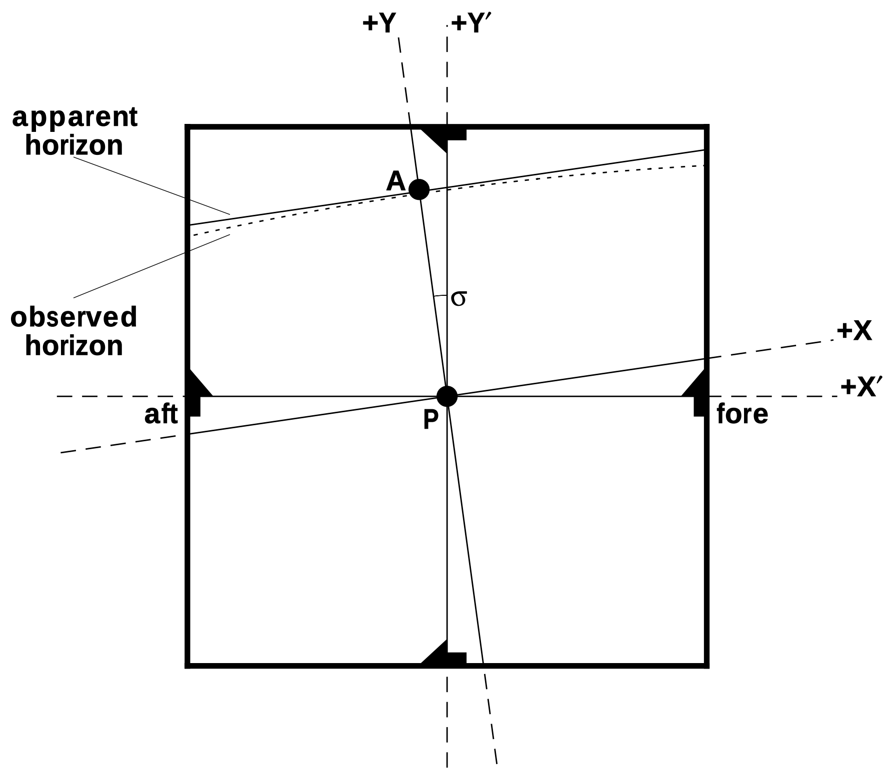


Figure 2. Coordinate systems for the analysis of an oblique trimetrogon image. The standard fore and aft fiducial marks define the $X'$-$Y'$ coordinate frame, centred at $P$. The principal point $P$ and the apparent horizon define the $X$-$Y$ coordinate frame.

The *apparent depression angle* $\theta_a$ is the angle between the optical axis of the camera and the plane of the apparent horizon. Trigonometry shows that $\tan\theta_a = AP/f$, where $f$ is the focal length of the camera. ($f$ is also equal to $OP$, the distance between $P$ and the camera's *perspective centre* $O$; Figure 3.) The *principal depression angle* $\theta_p = \theta_a + \alpha$, where $\alpha$ is the *dip angle*

$$\tan\alpha = m\sqrt{\delta(2+\delta)}. \qquad (1)$$

In (1), $m = 0.9216$ is a correction for atmospheric refraction and $\delta = h/R_E$, where $h$ is the flying height and $R_E = 6\ 371\ 023\,\mathrm{m}$ is the radius of the Earth (Imhof and Doolittle 1966). $\alpha$ is small by comparison with $\theta_a$, so here we accept the published estimate of $h$, which is treated later as an unknown to be solved for. Finally the *nadir* of the photograph, $N$, in the $X$-$Y$ frame, is located at $(x, y) = (0, y_N)$, where $y_N = -f \tan \tau_p$ and $\tau_p = 90° - \theta_p$ is the *principal tilt angle*.

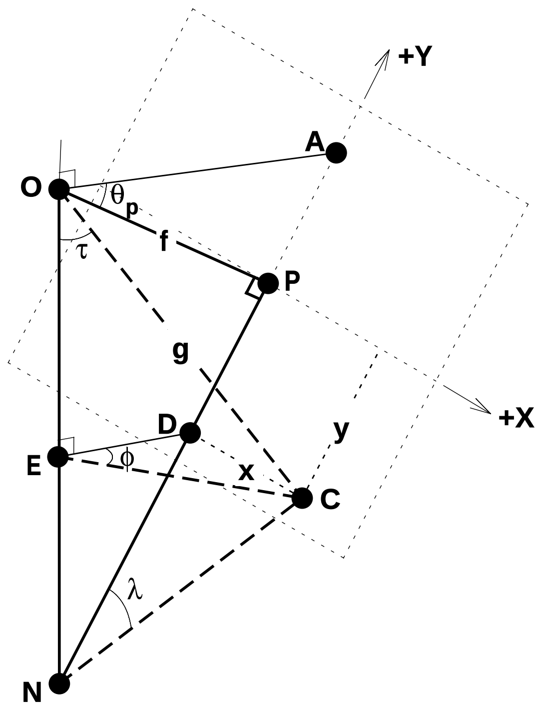


Figure 3. Relationships between image plane $PNC$, principal plane $OPN$ and image-point plane $OCN$. Both swing angle and dip angle are assumed to be zero for clarity. The photograph (dotted rectangle) lies in the image plane and may be visualized as a (positive) print located $f$ mm in front of the camera ($O$) along the camera's optical axis ($OP$); the photograph is perpendicular to the optical axis. $OA$ lies in the horizontal plane, and $ON$ is the local vertical. Any image point $C = (x, y)$ defines an *image-point plane*, which meets the principal plane in $ON$. The angle $\phi$ between these two (vertical) planes, and the tilt angle $\tau$ of the image point, are obtained by solving the triangle $CDE$, which is horizontal (that is, $DE$ is parallel to $AO$). Point $D$ lies in the image plane at $(0, y)$.

## Image Rectification Model

If an unknown point $(\xi, \eta)$ on the ground is imaged at point $(x, y)$ on the photograph, it may be estimated from the following model:

$$\xi = (b_0 - \zeta) \tan \tau \sin(\phi - b_1) + b_2, \qquad (2a)$$

$$\eta = (b_0 - \zeta) \tan \tau \cos(\phi - b_1) + b_3, \qquad (2b)$$

where $\zeta$ is the elevation of the ground point, $\tau$ is the tilt angle of the image point, and $\phi$ is the horizontal angle made at the nadir line $ON$ between the principal plane $OPN$ and the plane $OCN$ (see Figure 3 for details). The $b_i$ are the four unknown model parameters and are to be estimated.

For any image point $C$, both $\tau$ and $\phi$ may be calculated using the geometric information derived in the previous subsection. First we calculate the angle $\lambda = \angle PNC$ in the plane of the oblique photograph (Figure 3):

$$\tan \lambda = x/(y - y_N), \qquad (3)$$

$y_N$ being the $Y$ coordinate of the nadir $N$. We obtain $\phi$ from

$$\tan\phi = \tan\lambda/\sin\theta_p = \tan\lambda\sec\tau_p. \tag{4}$$

If $x = 0$ then $C$ lies in the principal plane and $\phi = 0$; the tilt angle $\tau$ is

$$\tau = \tau_p - \arctan(y/f). \tag{5}$$

When $x \neq 0$, the tilt angle is given by

$$\sin\tau = CE/g, \tag{6}$$

in which $g$ is the distance from perspective centre to image point

$$g = \sqrt{f^2 + x^2 + y^2} \tag{7}$$

and point $E$ is the point on the nadir line $ON$ defined by

$$CE = x/\sin\phi. \tag{8}$$

Putting $\psi = \phi - b_1$ and $\gamma = (b_0 - \zeta)\tan\tau$, equations 2 can be simplified as

$$\xi = \gamma\sin\psi + b_2, \tag{9a}$$
$$\eta = \gamma\cos\psi + b_3. \tag{9b}$$

Among the four unknown parameters of the model (Figure 4), $b_0$ is the elevation of the camera, for which reported flying height $h$ is a first guess. $(b_2, b_3)$ is the location of the nadir in the ground coordinate frame (i.e. that of $\xi$ and $\eta$), and $b_1$ is the angle measured at $(b_2, b_3)$ between the $X$-$Y$ and $\xi$-$\eta$ frames.

Notice that for each point, whether a control point ($\xi$ and $\eta$ known) or an arbitrary image point ($\xi$ and $\eta$ unknown), it is necessary to supply at least moderately accurate estimates of the elevation $\zeta$. This requirement might be relaxed for terrain of low relief, but in the region of Figure 1 the range of elevations exceeds 1 000 m and it is essential that $\zeta$ be supplied. (An alternative would be to recast the model to make elevation a third unknown quantity to be estimated at each point.)

The locations $(\xi, \eta, \zeta)$ of the control points are obtained from an appropriate source, such as a base map or a ground survey. The parameters $b_i$ are estimated with a non-linear least-squares inversion algorithm applied to equations 9, using the known control-point locations $(\xi, \eta, \zeta)$ on the ground and $(x, y)$ on the image. First guesses must be supplied for each parameter.

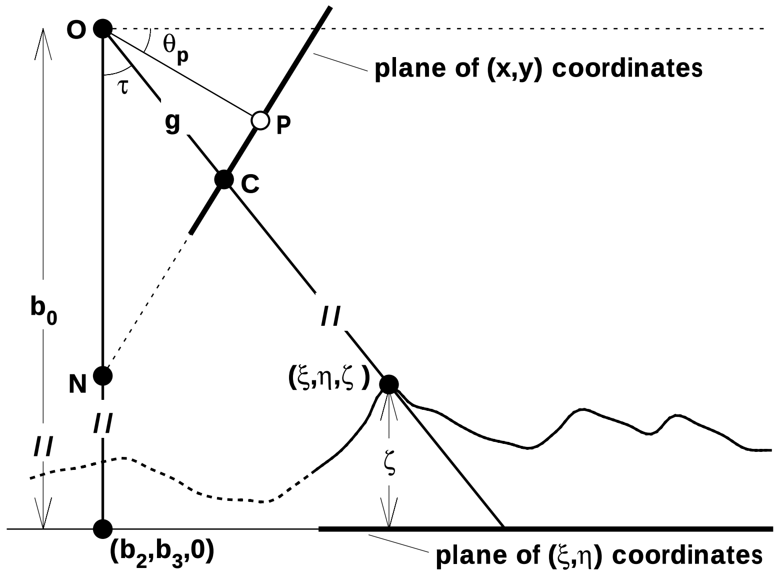


Figure 4. Geometry of the model parameters in the image-point plane $OCN$. (Note that in general $P$ does not lie in this plane.) $b_1$ is the angle between the $X$-$Y$ coordinate frame and the $\xi$-$\eta$ coordinate frame and is not visible in the image-point plane (but see Figure 5).

Finally the best-fit estimates of the parameters are used to run the model forwards so as to rectify the digitized outlines of features of interest on the image. That is, equations 9, with the $b_i$ now known, are used to estimate the ground locations $(\xi, \eta)$ of digitized image points $(x, y, \zeta)$ representing such features as glacier terminuses.

## APPLICATION AND RESULTS

We estimated the swing angle $\sigma$ as $3.39 \pm 0.14°$, the dip angle $\alpha$ as $2.42 \pm 0.10°$ and the depression angle $\theta_p$ as $30.96 \pm 0.06°$. (Error ranges are twice the standard error; they derive from an estimate, based on repeated trial measurements, of 0.1 mm for the standard error of any distance measured on photograph or map, and also for the focal length of the camera.)

For photograph 60LT-8M219 (Figure 1), the focal length $f$ of the camera is 154.2 mm (6 in) and the nominal flying height $h = 6\ 706$ m (22 000 ft). As first guesses at the parameters we chose $b_0^{(0)} = h$ and estimated $b_1^{(0)}$ and $[b_2^{(0)}, b_3^{(0)}]$ by comparison of the vertical photograph with the base map. We digitized features of interest, and 14 control points (Figure 5), from the photograph.

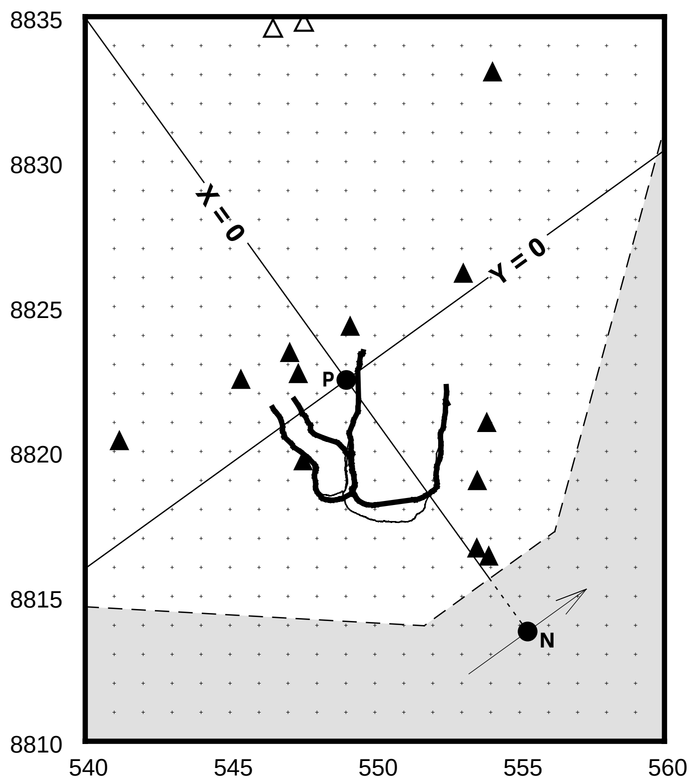


Figure 5. Outlines of the terminuses of White Glacier and Thompson Glacier, Axel Heiberg Island, photogrammetrically rectified from an image taken in 1948 (left-oblique photograph 60LT-8M219; thick lines) and digitized from a map based on vertical photography taken in 1960 (National Research Council 1962b; thin lines). The map was drawn on the basis of geodetic control measurements on the ground. Control points for the 1948 photogrammetry are shown as triangles, the two open triangles representing control points discarded during model fitting. The unshaded area is an approximate representation of the field of view of the photograph. The projections onto the $\xi$-$\eta$ coordinate plane of the axes of the $X$-$Y$ coordinate frame (making an angle of $b_1 \simeq 36°$ with the $\xi$-$\eta$ frame), the nadir $N$ (where $(\xi, \eta) = (b_2, b_3)$) and the principal point $P$ are also shown. The arrow passing through $N$ represents the aircraft flight path. The $(\xi, \eta)$ coordinates (km) are those of UTM zone 15.

The locations $(\xi, \eta, \zeta)$ of the control points were read from a 1:50 000 map of the glacier terminus region (National Research Council, 1962a). The contour interval was 25 m, and we assumed a standard error of 10 m for $\zeta$. Most control points were hilltops, but not all of the ground survey stations described by Haumann (1963) could be located. The two control points most distant from the camera were discarded; they detracted noticeably from the quality of fit of the model, probably because we could not identify them with sufficient accuracy on the photograph. The remainder, however, surrounded the glacier terminuses.

**Table 1 — First-guess and Best-fit Estimates of Model Parameters**

| *Parameter* | *First-guess* | *Best-fit* |
|---|---|---|
| $b_0$ | 6 706 m | 6 839 m |
| $b_1$ | 36.06° | 35.80° |
| $b_2$ | 555 266 m | 555 275 m |
| $b_3$ | 8 813 891 m | 8 813 787 m |

All the best-fit estimates for the parameters were near the corresponding first guesses; for example that for $b_0$, flying height, was 6 839 m, or 123 m above the nominal height (Table 1). The standard errors estimated for the parameters were all small, and the fit was very satisfactory. The root-mean-square residuals of the control points ranged from 16 m up to 207 m; that is, the model estimates of control-point location were up to about 200 m away from the corresponding actual locations in the local ($\xi$-$\eta$) coordinate system.

## CONCLUSION

It emerges from the photogrammetric work done on Figure 1 that in 1948 White Glacier was significantly further forward, and Thompson Glacier significantly further back, than in 1960, the earliest date for which Cogley *et al.* (1996) were able to map the terminus positions. Cogley and Adams (1999) calculate that between 1948 and 1960 White Glacier retreated at $-12.6\pm1.2$ m a$^{-1}$ while Thompson Glacier advanced at $+58.5\pm2.0$ m a$^{-1}$. This is not the place to enlarge upon the glaciological meaning of these contrasts in behaviour of two contiguous terminuses. The numbers are mentioned simply to illustrate the precision obtainable with the algorithm described above and to highlight the potential of careful photogrammetric analysis in the study of historical changes in the cryosphere.

## REFERENCES

Cogley, J.G., M.A. Ecclestone and W.P. Adams, 1996, Fluctuations of the terminuses of White and Thompson Glaciers, Axel Heiberg Island, N.W.T., Canada, *Eastern Snow Conference Proceedings*, **53**, 83-94.

Cogley, J.G., and W.P. Adams, 1999, Photographic resources for monitoring glacier fluctuations on Axel Heiberg Island, *Arctic*, **52**, submitted.

Dunbar, M.J., and K.R. Greenaway, 1956, *Arctic Canada from the Air*. Queen's Printer, Ottawa, Ontario. 541p.

Haumann, D., 1963, Surveying glaciers in Axel Heiberg Island, *Canadian Surveyor*, **17**, 81-95.

Imhof, R.K., and R.C. Doolittle, 1966, Mapping from oblique photographs, in Thompson, M.M., ed., *Manual of Photogrammetry*, 3rd ed., 875-917. American Society of Photogrammetry, Falls Church, Virginia.

National Research Council, 1962a, *Thompson Glacier Region, Axel Heiberg Island, N.W.T., Canada*. Map at 1:50 000. Photogrammetric Research Section, National Research Council of Canada, Ottawa, in conjunction with Axel Heiberg Island Expedition, McGill University, Montreal.

National Research Council, 1962b, *Thompson Glacier Snout, Axel Heiberg Island, Canadian Arctic Archipelago*. Map at 1:5 000. Photogrammetric Research Section, National Research Council of Canada, Ottawa, in conjunction with Axel Heiberg Island Expedition, McGill University, Montreal.